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Into Mesh Lubrication of Spur Gears With Arbitrary Offset Oil Jet
II—For Jet Velocities Equal to or Greater Than Gear Velocity

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INTO MESH LUBRICATION OF SPUR GEARS WITH ARBITRARY OFFSET OIL JET

II - FOR JET VELOCITIES EQUAL TO OR GREATER THAN GEAR VELOCITY

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ABSTRACT

An analysis was conducted for into mesh oil jet lubrication with an arbitrary offset and inclination angle from the pitch point for the case where the oil jet velocity is equal to or greater than gear pitch line velocity. Equations were developed for minimum and maximum oil jet impingement depth. The analysis also included the minimum oil jet velocity required to impinge on the gear or pinion and the optimum oil jet velocity required to obtain the best lubrication condition of maximum impingement depth and gear cooling. It was shown that the optimum oil jet velocity for best lubrication and cooling is when the oil jet velocity equals the gear pitch line velocity. When the oil jet velocity is slightly greater than the pitch line velocity the loaded side of the driven gear and the unloaded side of the pinion receive the best lubrication and cooling with slightly less impingement depth. As the jet velocity becomes much greater than the pitch line velocity the impingement depth is considerably reduced and may completely miss the pinion.

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INTRODUCTION

In the lubrication and cooling of gear teeth a variety of oil jet lubrication schemes are sometimes used. A method commonly used is a low pressure low velocity oil jet directed at the ingoing mesh of the gears as was analysed in (1). Sometimes an oil jet is directed at the outgoing mesh at low pressures. It was shown in (2) that the out of mesh lubrication method provides a minimal impingement depth and low cooling of the gears due to the short fling off time and fling off angle (3). In (4, 5) it was shown that a radially directed oil jet near the out of mesh position with the right oil pressure was the method that provided the best impingement depth. Reference 6 showed this to give the best cooling. However, there are still many cases where into mesh lubrication is used with low oil jet pressure which does not provide the optimum oil jet penetration and cooling. It should also be noted that excessive into mesh lubrication can cause high losses in efficiency from gear churning and trapping in the gear teeth (7). In (8) the case for into mesh lubrication with oil jet velocity equal to or less than pitch line velocity was analyzed and equations developed for impingement depth for several jet velocities.

The objective of the work reported herein was to develop the analytical methods for gear lubrication with the oil jet directed into mesh and with the oil jet velocity equal to or greater than the pitch line velocity. When the oil jet velocity is greater than the pitch line velocity for into mesh lubrication the impingement depth is determined by the trailing end of the jet after it has been cut off or chopped by the following tooth. The analysis is therfore somewhat different than Part I of this paper (8) for the case where the oil jet velocity is less than the pitch line velocity. The oil jet location should be offset from the pitch point with an inclined

angle to obtain optimum cooling of both gear and pinion for other than one to one gear ratios. The analysis presented herein assumes an arbitrary offset and inclination angle to obtain an optimum oil jet velocity for various gear ratios. Further analysis is needed to determine the optimum offset and inclination angle for various gear ratios.

ANALYSIS

The high speed cooling jet conditions discussed in this analysis are used only when a range of duty cycle conditions dictate a wide operating speed range with a constant oil jet velocity that must be suitable over the whole range of speeds. Starting with Figure 1 the sequence of events for the pinion in the case where $V_j > \omega_p r \sec \beta_p$ is shown in Figures 1 through 4. Here, instead of tracking the head of the jet stream as in Part I of this paper (8), the trailing end or "tail" of the stream will be tracked after it is chopped by the gear tooth (1). This is shown at "A" in Figure 3 to the final impingement at a depth "d_p" on the pinion tooth (2) as shown in Figure 4. Initial impingement on the pinion starts as the pinion top land leading edge crosses the jet stream line with inclination angle set at β_p and offset S_p as shown in Figure 1.

The position of the pinion at this time is θ_{p3}, defined as (from Figure 1):

$$\theta_{p3} = \cos^{-1}(r_s/r_o) - \text{inv}\,\varphi_{op} + \text{inv}\,\varphi \tag{1}$$

where:

$$\text{inv}\,\varphi_{op} = \tan\varphi_{op} - \varphi_{op} \quad \text{and}$$

$$\varphi_{op} = \cos^{-1}(r_b/r_o)$$

$$\text{inv}\,\varphi = \tan\varphi - \varphi$$

Generally the arbitrarily set offset "S" for the gear establishes the value of β_p from:

$$\beta_p = \tan^{-1}[S/(R_o^2 - R_s^2)^{1/2}] \qquad (2)$$

Given β_p, then s_p can be calculated from

$$S_p = [(r_o^2 - r^2 \cos^2 \beta_p)^{1/2} + r \sin \beta_p] \sin \beta_p \qquad (3)$$

so that:

$$r_s = r - S_p$$

The tail of this jet stream is finally chopped at "A" in Figure 3 by the gear top land leading edge. The position of the gear at this time is θ_{g1} calculated:

$$\theta_{g1} = \cos^{-1}(R_s/R_o) - \text{inv}\ \varphi_{og} + \text{inv}\ \varphi \qquad (4)$$

where:

$$\text{inv}\ \varphi_{og} = \tan \varphi_{og} - \varphi_{og} \quad \text{and}$$

$$\varphi_{og} = \cos^{-1}(R_b/R_o)$$

Then the position of the pinion at time equal to zero (t = 0) is calculated from (see Figure 3):

$$\theta_{p4} = m_g \theta_{g1} + \text{inv}\ \varphi \qquad (5)$$

which locates the pinion at the time of the flight of the tail of the jet stream when it is initiated. The jet tail continues to approach the trailing side of the pinion tooth profile until it reaches the position shown in Figure 4 when it terminates at time $t = t_f$. The position of the pinion at this time is calculated from (see Figure 4):

$$\theta_{p5} = \tan^{-1}\left(\frac{L_p \cos \beta_p}{r - L_p \sin \beta_p}\right) + \text{inv}\ \varphi_{p5} \qquad (6)$$

where:

$$\text{inv}\ \varphi_{p5} = \tan \varphi_{p5} - \varphi_{p5}$$

$$\varphi_{p5} = \cos^{-1}\left(\frac{r_b}{[(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2}}\right) \tag{7}$$

The design solution to the problem of pinion cooling when "d_p" is specified is to solve for the jet V_j explicitly based on the fact that $t_f = t_\omega$ as shown in Part I of this paper. Thus, the required jet velocity is calculated from:

$$V_j = \frac{[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p]\omega_p}{\theta_{p4} - \theta_{p5}} \tag{8}$$

where:

$$L_p = [(r_o - d_p)^2 - r^2 \cos^2 \beta_p]^{1/2} + r \sin \beta_p \tag{9}$$

The analysis solution to the problem when V_j is specified $[V_j(\text{Opt, U})_p < V_j < \infty]$ so that the resulting impingement depth d_p, can be calculated implicitly by solving iteratively for "L_p" from (Figure 1):

$$\omega_p[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p] = (\theta_{p4} - \theta_{p5})V_j \qquad \text{then} \tag{10}$$

$$d_p = r_o - [(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2} \tag{11}$$

Moving up along the velocity scale of Table 1 from $\omega_p r \sec \beta_p = V_j$, then it can be shown that the upper limit $V_j(\text{Opt, U})$ for the "constant impingement depth range" where $d_p = a$, can be calculated for the pinion from:

$$V_j(\text{Opt, U})_p = \omega_g(R_o^2 - R_s^2)^{1/2} \sec \beta_p/\theta_{g1} \tag{12}$$

Thus, if the jet velocity V_j is between the lower limit $V_j(\text{Opt}, L)_p < V_j < V_j(\text{Opt}, U)_p$ then the impingement depth will be $d_p = a = 1/P_d$ on at least one side of the tooth profile. If $V_j = \omega_p r \sec \beta_p$ exactly then <u>$d_p = a$ on both sides of the tooth profile</u>. Increasing the jet velocity above $V_j(\text{Opt}, U)_p$ reduces the impingement depth d_p until at $V_j(\max)_p$ the tail of the jet chopped by the gear tooth is moving so fast as to be just missed by the pinion top land leading edge "A" in Figure 6 when $m_g > m_g(\text{crit})$. The upper limit critical gear ratio, as a function of N_p and assuming $V_j(\max)_p = \infty$, may be calculated from:

$$m_g(\text{crit}) = \frac{\cos^{-1} \dfrac{N_p'}{N_p + 2} - \text{inv}\left(\cos^{-1} \dfrac{N_p \cos \varphi}{N_p + 2}\right) + \text{inv } \varphi + \pi/N_p - 2\, B_p/N_p}{\cos^{-1}\left(\dfrac{m_g(\text{crit})N_p + 2P_dS}{m_g(\text{crit}) + 2}\right) - \text{inv } \cos^{-1}\left(\dfrac{N_p \cos \varphi}{N_p + 2/m_g(\text{crit})}\right) + \text{inv } \varphi} \tag{13}$$

where:

$$N_p' = [N_p \cos^2 \beta_p - ((N_p + 2)^2 - N_p^2 \cos^2 \beta_p)^{1/2} \sin \beta_p] \tag{14}$$

and

$$\beta_p = \tan^{-1}\left\{S\, P_d/[m_g(\text{crit})N_p(1 - P_dS)^2 - (P_dS)^2]^{1/2}\right\} \tag{15}$$

and

$$P_dS = \frac{(S/S_o)[m_g(\text{crit}) - 1]}{[m_g(\text{crit}) + 1]} \tag{16}$$

when $S = S_o$ and $\beta = \beta_{pp}$, $V_j(\max)_p < \infty$, the $m_g(\text{crit})$ ceases to exist.

When the maximum jet stream velocity ($V_j = V_j(\max)_p$) is reached, the initial position of the pinion as the gear tooth chops the tail of the jet stream may be calculated from (see Figure 5):

$$\theta_{p6} = m_g\theta_{g1} - \pi/N_p - \text{inv } \varphi + \text{inv } \varphi_{op} + 2\, B_p/N_p \tag{17}$$

and the final position at the point "A" in Figure 6 is calculated from:

$$\theta_{op} = \cos^{-1}\left(\frac{r_s}{r_o}\right)$$

The maximum jet velocity may then be calculated from:

$$V_j(\max)_p = \frac{\omega_p[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - (r_o^2 - r_s^2)^{1/2} \sec \beta_p]}{\theta_{p6} - \theta_{op}} \tag{18}$$

Therefore, when $V_j \geq V_j(\max)_p$, $d_p = 0$, if $m_g > m_g(\text{crit})$. Also if $S = S_o$ and $\beta = \beta_p = \beta_{pp}$, then $V_j(\max)_p \to \infty$. $V_j(\max)_p = V_j(\text{Opt, U})$ then $V_j(\max)_p$ is set equal to $V_j(\text{Opt, U})$. Stated differently, when $V_j(\text{Opt, U})$ is greater than or equal to the calculated $V_j(\max)_p$ then $V_j(\max)_p$ no longer physically represents the solution and $V_j(\text{Opt, U})$ is the maximum value that can be allowed for V_j.

Again, it should be noted that the selection or specification for V_j must be kept within the bounds of $\omega_p r \sec \beta_p$ and Equation (18) if impingement on the trailing side of the tooth profile is desired.

The Equations (8) through (18) have been summarized on Table 1 on a velocity scale to add graphic visibility to their usability range.

<u>The Gear – When the Jet Velocity is Greater than Pitch Line Velocity ($V_j > \omega_g R \sec \beta_p$)</u>

The sequence of events for the gear in the case where $V_j > \omega_g R \sec \beta_p$ is shown in Figures 5, 7, and 8. Again, instead of tracking the head, the trailing end or "tail" of the jet stream will be tracked after it is chopped at time ($t = 0$) as shown at "A" in Figure 7 to the final impingement point at a depth "d_g" at time $t = t_\omega$ as shown in Figure 8. The position of the pinion at time ($t = 0$) may be calculated from θ_{p3}, defined above. The associated gear position can be calculated from:

$$\theta_{g6} = (\theta_{p3}/m_g) \cdot \text{inv}\ \varphi \tag{19}$$

which locates the gear at the time ($t = 0$) when the flight of the tail of the jet stream is initiated (see Figure 7). The position of the gear when the jet stream tail is terminated on the gear may be calculated from:

$$\theta_{g7} = \tan^{-1}\left(\frac{L_g \cos \beta_p}{R + L_g \sin \beta_p}\right) + \text{inv}\ \varphi_{g7} \tag{20}$$

where

$$\text{inv}\ \varphi_{g7} = \tan \varphi_{g7} - \varphi_{g7}$$

$$\varphi_{g7} = \cos^{-1}\left(\frac{R_b}{[(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2}}\right) \tag{21}$$

as shown in Figure 8 at time ($t = t_\omega$).

Once again, when $0 \le \beta_p < \beta_{pp}$ and $0 \le S < S_o$; the analysis solution to the problem of cooling the gear is constrained by the jet velocity limits for the pinion to maintain impingement on same. And, as explained above, a given "gear mesh" must have a common jet velocity. Accordingly, a given impingement depth is selected for the pinion. Then, the associated jet velocity V_{jp} is solved for this velocity which can then be used to find the associated gear impingement depth "d_g". Thus, after finding V_{jp}, solve for L_g iteratively from:

$$[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - L_g]\omega_g = (\theta_{g6} - \theta_{g7})V_{jp} \tag{22}$$

and

$$d_g = R_o - [(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2} \tag{23}$$

for $V_j(\text{Opt}, U)_p < V_j < V_j(\max)_p$.

The design solution to the problem when $V_j(\text{min})_g < V_j < V_j(\text{max})_g$ may be calculated from:

$$V_j = \frac{[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - ((R_o - d_g)^2 - R^2 \cos^2 \beta_p)^{1/2} + R \sin \beta_p]\omega_g}{\theta_{g6} - \theta_{g7}} \quad (24)$$

with the additional restriction that $V_j(\text{Opt}, U)_g < V_j < V_j(\text{max})_g = V_j(\text{max})_p$. As for the others, Equation (24) is shown placed on the velocity scale of Table 2.

Also if V_j is specified within the range allowed for V_{jp} for Equation (8) and (24), then d_g can be calculated implicitly by solving iteratively for "L_g" from:

$$[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - L_g]\omega_g = (\theta_{g6} - \theta_{g7})V_{jp} \qquad \text{and} \qquad (25)$$

$$d_g = R_o - [(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2} \quad (26)$$

<u>When the Jet Velocity is Equal to Pitch Line Velocity ($v_j = \omega_g R \sec \beta_p$)</u>

Continuing up the velocity scale of Table 2 from $V_j = \omega_g R \sec \beta_p$ it can be shown that the upper limit for the constant impingement depth range, where $d_g = a$, can be calculated for the gear from:

$$V_j(\text{Opt}, U)_g = \omega_p[(r_o^2 - r_s^2)^{1/2} \sec \beta_p] / \theta_{p3} \quad (27)$$

Thus, if the jet velocity V_j is between $V_j(\text{Opt}, L)_g < V_j < V_j(\text{Opt}, U)_g$, the impingement depth will be $d_g = a = 1/P_d$ on at least one side of the gear tooth profile. If $V_j = \omega_g R \sec \beta_p$ exactly, the $d_p =$ "a" on both sides of the tooth profile.

Increasing the jet velocity above $V_j(\mathrm{Opt}, U)_g$ reduces the impingement depth d_g until $V_j = V_j(\max)_g$. When $S < S_o$, the tail of the jet chopped by the gear tooth is moving so fast as to be just missed by the pinion top land so that $d_p = 0$ and $m_g = m_g(\mathrm{lim})$. When $S = S_o$ and $V_j(\max)_g \to \infty$, then $d_p \to 0$.

The initial position of the gear when it chops the tail of the leading jet stream is θ_{g1} as defined above when $m_g > m_g(\mathrm{crit})$ and as shown in Figure 5 (for S = 0). The limit position at "A" in Figure 6 as the jet tail just misses the pinion top land is calculated from:

$$\theta_{g8} = (\theta_{p3}/m_g) + \pi/N_g + 2\, B_g/N_g \tag{28}$$

$V_j(\max)_g$ in gear parameters may be calculated from:

$$V_j(\max)_g = \frac{\omega_g[(R_o^2 - R_s^2)^{1/2} - (r_o^2 - r_s^2)^{1/2}] \sec \beta_p}{\theta_{g1} - \theta_{g8}} \quad (m_g \neq 1) \tag{29}$$

Note that as $V_j(\max)_g \to \infty$ for $m_g \leq m_g(\mathrm{crit})$ and when $m_g > m_g(\mathrm{crit})$ then $V_j(\max)_g$ is finite at $d_p = 0$.

The impingement distance $L_g(\max)$ when $V_j = V_j(\max)_g$ may be iterated from:

$$\omega_g[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - L_g(\max)] = (\theta_{g6} - \theta_{g7}) V_j(\max)_g \tag{30}$$

Then when $S < S_o$

$$d_g = d_g(\min, U) = R_o - \{[R + L_g(\max) \sin \beta]^2 + [L_g(\max) \cos \beta]^2\}^{1/2} \tag{31}$$

If $V_j(\max)_g \leq V_j(\mathrm{Opt}, U)$ then set $V_j(\max)_g$ equal to $V_j(\mathrm{Opt}, U)$. Also, it should be observed that since only one V_{jp} can be used, we must set Equations (18) and (29) equal: $V_j(\max)_p = V_j(\max)_g$ for the given m_g, making the design $m_g = m_g(\mathrm{lim})$ when $m_g > m_g(\mathrm{crit})$.

SUMMARY

An analysis was conducted for into mesh oil jet lubrication with an arbitrary offset and inclination angle from the pitch point for the case where the oil jet velocity is equal to or greater than pitch line velocity. Equations were developed for minimum and maximum oil jet impingement depths. The equations were also developed for the maximum oil jet velocity allowed so as to impinge on the pinion and the optimum oil jet velocity required to obtain the best lubrication condition of maximum impingement depth and gear tooth cooling. The following results were obtained:

1. The optimum operating condition for best lubrication and cooling is exactly provided when the jet velocity is equal to pitch line velocity $V_j = V_g \sec \beta_p = \omega_p r \sec \beta_p = \omega_g R \sec \beta_p$ whereby both sides of the pinion and gear will be wetted and the maximum impingement depth to the pitch line is obtained.
2. When the jet velocity is slightly greater than the pitch line velocity $\omega_p r \sec \beta_p < V_j < V_j(\mathrm{Opt}, U)$ the loaded side of the driven gear is favored and receives the best cooling with a slightly less oil impingement than when $V_j = \omega_p r \sec \beta_p$.
3. As the jet velocity becomes much greater than the pitch line velocity $V_j(\mathrm{Opt}, U) < V_j < V_j(\max)_p$ the impingement depth is considerably reduced. As a result, the pinion may be completely missed by the lubricant so that no direct cooling of the pinion is provided when $V_j(\max)_p \le V_j$.

NOMENCLATURE

a	$1/P_d$ or $(1 \pm \Delta\ N/2)/P_d$ = addendum
b_p, b_g	pinion and gear backlash respectively
B_p, B_g	total, pinion, gear backlash at $P_d = 1$
d_p, d_g	radial impingement depth
b_p, b_g	$2B_p/N_p$, $2B_g/N_g$
L_p, L_g	pinion, gear final impingement distance
L_{ig}	intermediate impingement distance
m_g	$N_g/N_p = R/r = \omega_p/\omega_G$ = gear ratio
N_p, N_g	number of teeth in pinion, gear
ΔN	differential number of teeth
P_d	diametral pitch
r, R	pinion and gear pitch radii
r_α, R_α	perpendicular distance from pinion, gear center to jet line
r_s, R_s	distance along line of centers to jet line origin
r_x, R_x	distance along line of centers to jet line intersection at x
r_o, R_o	pinion and gear outside circle diameter
r_b, R_b	pinion and gear base radii
S, S_o, S_p	arbitrary jet nozzle offset to intersect O.D.'s. Offset for pinion only
t	time
t_f, t_ω	time of flight, rotation
$V_p = V_g$	linear velocity of pinion and gear at pitch line
V_j, V_{jp}	oil jet velocity, general, pinion controlled
x	distance from offset perpendicular to jet line intersection

$V_j(max)_p$	maximum velocity at which $d_p = 0$
$V_j(min)_p$	minimum velocity at which $d_p = 0$
β	arbitrary oil jet inclination angle
β_p	constrained inclination angle
β_{pp}	inclination angle for pitch point intersection
φ	pressure angle at pitch circles
φ_{pi}, φ_{gi}	pinion and gear pressure angle at points specified at i
ω_p, ω_g	pinion and gear angular velocities
inv φ	$\tan\varphi - \varphi$ = involute function at pitch point or operating pressure angle
$V_j(Opt, U)_p$	upper limit jet velocity to impingement at pitch line

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TABLE 1. – EQUATIONS FOR OIL JET VELOCITY AND PINION IMPINGEMENT DEPTH FOR PITCH LINE AND HIGHER OIL JET VELOCITY

Relative velocity scale	Oil jet velocity	Pinion Impingement Depth
Critical high velocity to miss the pinion	$V_j(\max)_p = \dfrac{*[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - (r_o^2 - r_s^2)^{1/2} \sec \beta_p]\, \omega_p}{\theta_{p6} - \theta_{op}}$ $V_j(\max)_p \to \infty$, when $s = s_o$ and $\beta_p = \beta_{pp}$	$d_p(\min, U) = 0^*$ $(m_g = 1)$ *when $m_g \geq m_g(\mathrm{crit})$ and $0 \leq s < s_o$ and $0 \leq \beta_p < \beta_{pp}$
Higher than pitch line velocity up to where the jet starts to miss the pinion	$V_j = \dfrac{\omega_p\{(R_o - R_s)^{1/2} \sec \beta_p - L_p\}}{\theta_{p4} - \theta_{p5}}$ V = given (when $0 \leq d_p \leq a$ only)	d_p = given (usual design solution and $L_p = [(r_o - d_p)^2 - r^2 \cos^2 \beta_p]^{1/2} + r \sin \beta_p$ Iterate L_p from: $[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p]\, \omega_p = (\theta_{p2} - \theta_{p1})V_j$ $d_p = r_o - [(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2}$
Slightly higher and pitch line-upper end of velocity plateau	$V_j(\mathrm{opt}, U)_p = \dfrac{\omega_g(R_o^2 - R_s^2)^{1/2} \sec \beta_p}{\theta_{g1}}$ $V_j = \omega_p r \sec \beta_p = \omega_g R \sec \beta_p$	$d_p(\mathrm{tail}) = a$, (trailing profile only) $d_p = a = (1 \pm \Delta N_p/2)/P_d$, (both profiles)

TABLE 2. - EQUATIONS FOR OIL JET VELOCITY AND GEAR IMPINGEMENT DEPTH FOR PITCH LINE AND HIGHER OIL JET VELOCITIES

Relative velocity scale	Oil jet velocity	Pinion Impingement Depth
Critical high velocity to miss the gear	$V_j(\max)_g^* = \dfrac{\omega_g[(R_o^2 - R_s^2)^{1/2} - (r_o^2 - r_o^2)^{1/2}]\sec\beta_p}{\theta_{g1} - \theta_{g8}}$ $= V_j(\max)_p \to \infty$, when $S = S_o$ and $\beta_p = \beta_{pp}$	$d_g(\min, U) = R_o - \{[R + L_g(\max)\sin\beta_p] + [L_g(\max)\cos\beta_p]^2\}^{1/2}$ *when $m_g \geq m_g(crit)$ <u>only</u> and $d_g(\min, u) = 0$ when $S = S_o$ and $\beta_p = \beta_{pp}$
Greater than pitch line velocity up to where the oil jet starts to miss the gear	$V_j = \dfrac{\omega_g\{(r_o^2 - r_s^2)^{1/2}\sec\beta_p - [(R_o - d_g)^2 - (R\cos\beta_p)^2]^{1/2} - R\sin\beta_p\}}{\theta_{g6} - \theta_{g7}}$ where $V_j(\min)_p \leq V_j \leq V_j(\max)_p$ V_j given	d_g given (usual design solution) $L = [(R_o - d_g)^2 - R^2\cos^2\beta_p]^{1/2} - R\sin\beta_p$ Iterate L_g from: $[(r_o^2 - r_o^2)\sec\beta_p - L_g]\omega_g = (\theta_{g1} - \theta_{g2})V_j$ then $d_g = R_o - [(R + L_g\sin\beta_p)^2 + (L_g\cos\beta_p)^2]^{1/2}$
Slightly higher and pitch line-upper end of velocity plateau	$V_j(opt, U)_g = \dfrac{\omega_p(r_o - r_s)^{1/2}\sec\beta_p}{\theta_{p3}}$ $V_j = \omega_g R\sec\beta_p(\sigma$ pitch point)	$d_g = a = \dfrac{1}{P_d} \pm \dfrac{\Delta N}{2P_d}$ (trailing profile) $d_g = a = \dfrac{(1 \pm \Delta N/2)}{P_d}$ (both profiles)

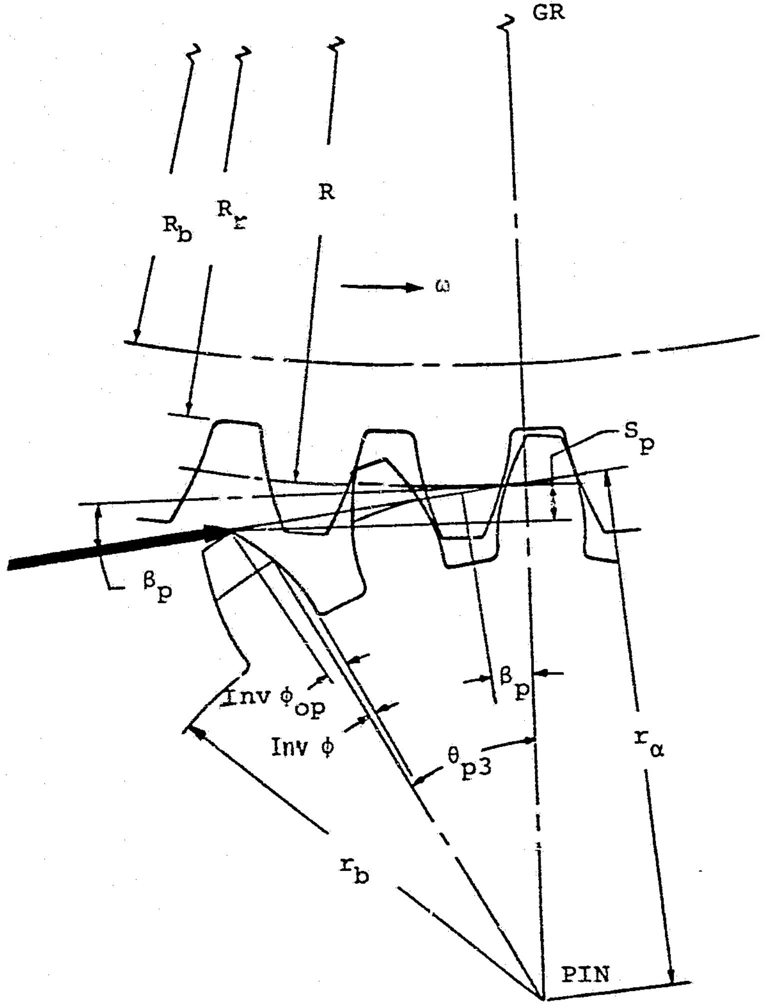

Figure 1.

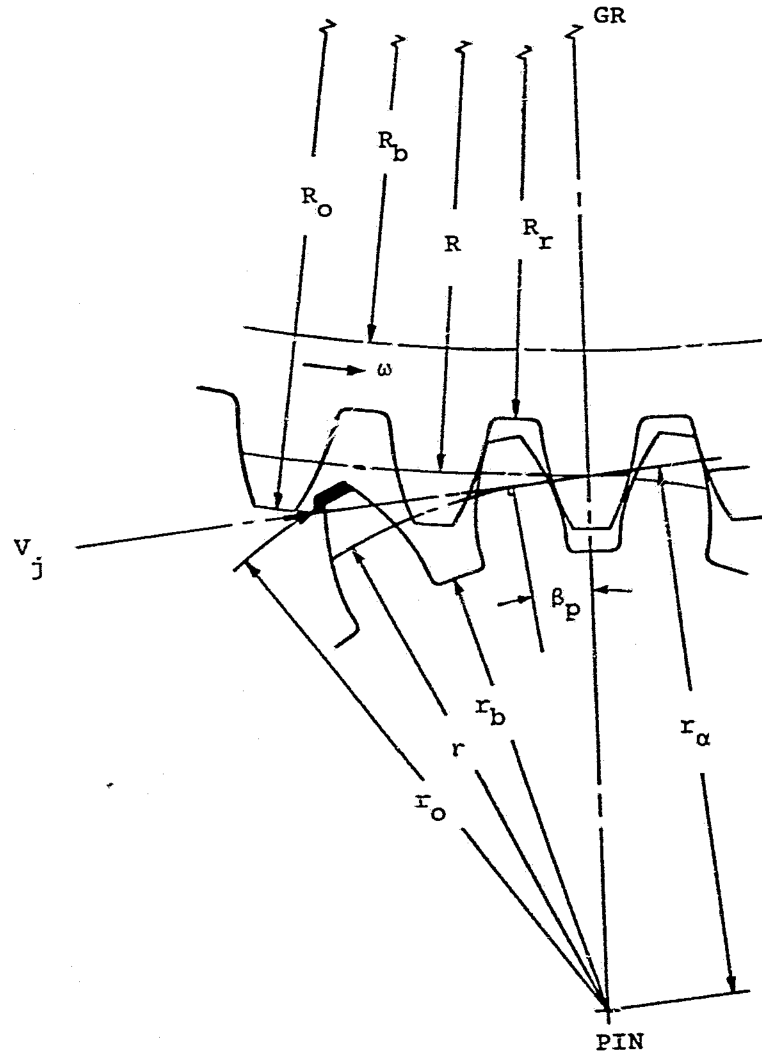

Figure 2.

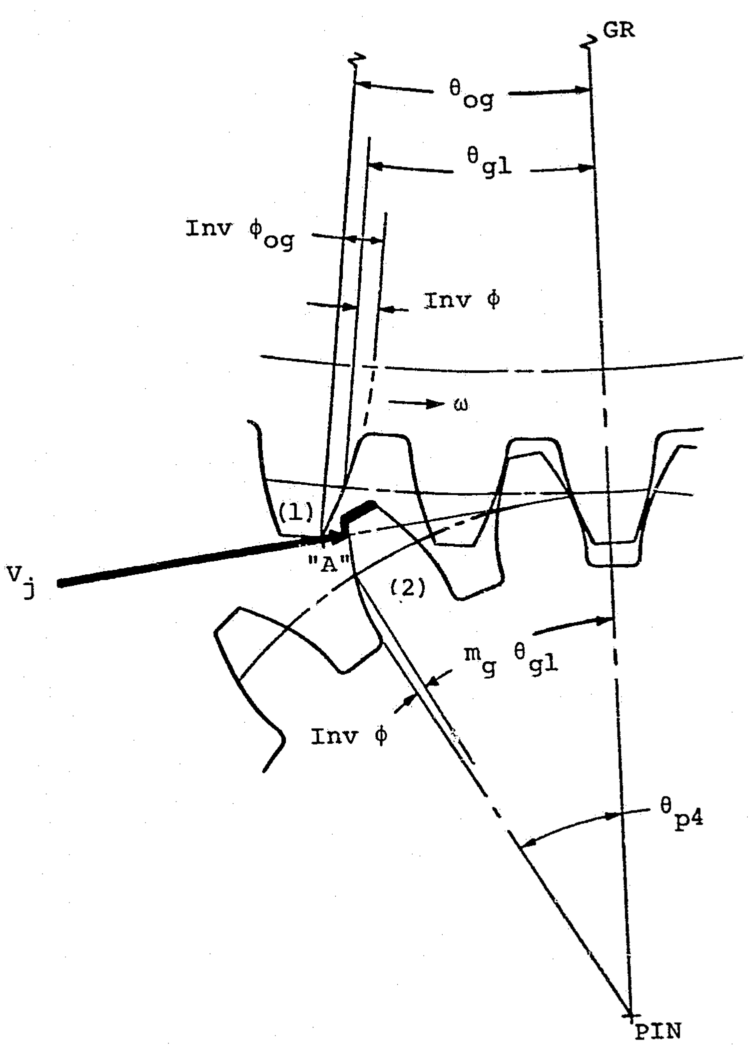

Figure 3.

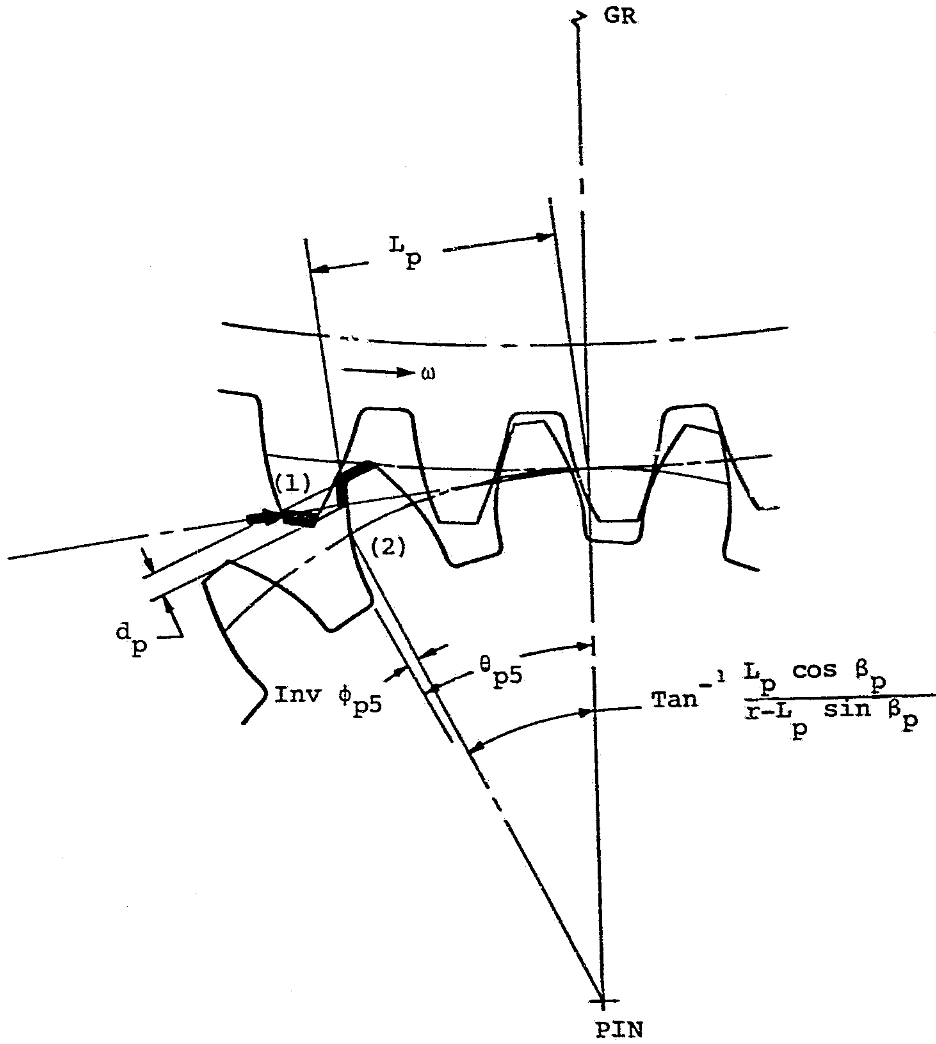

Figure 4.

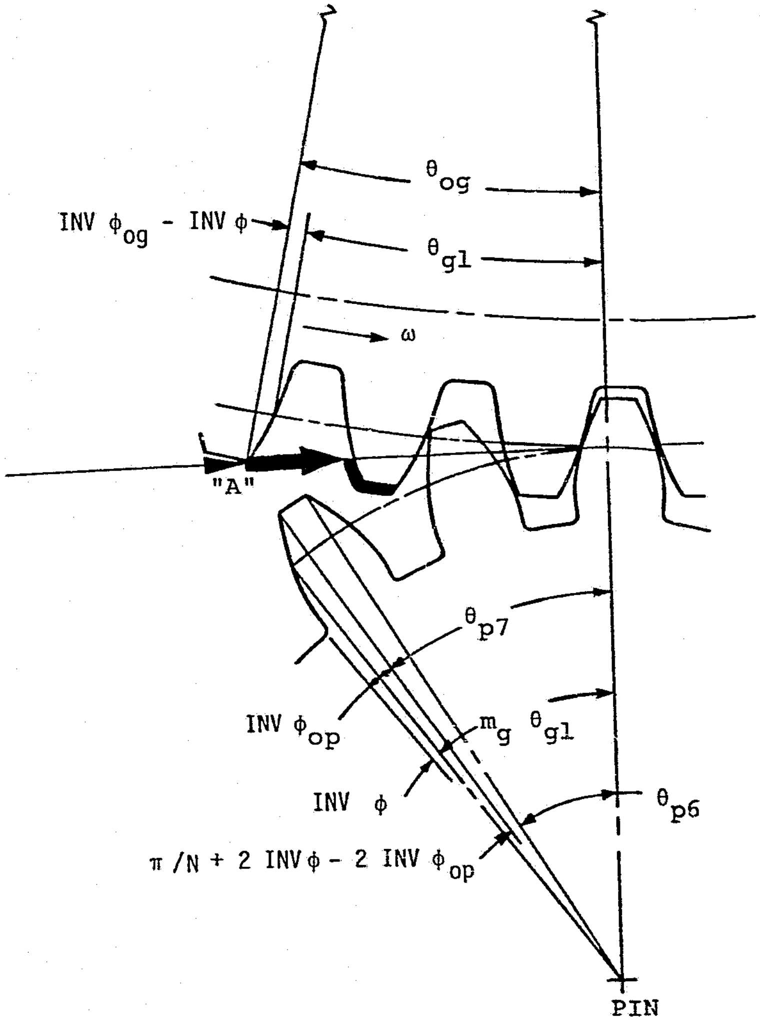

Figure 5.

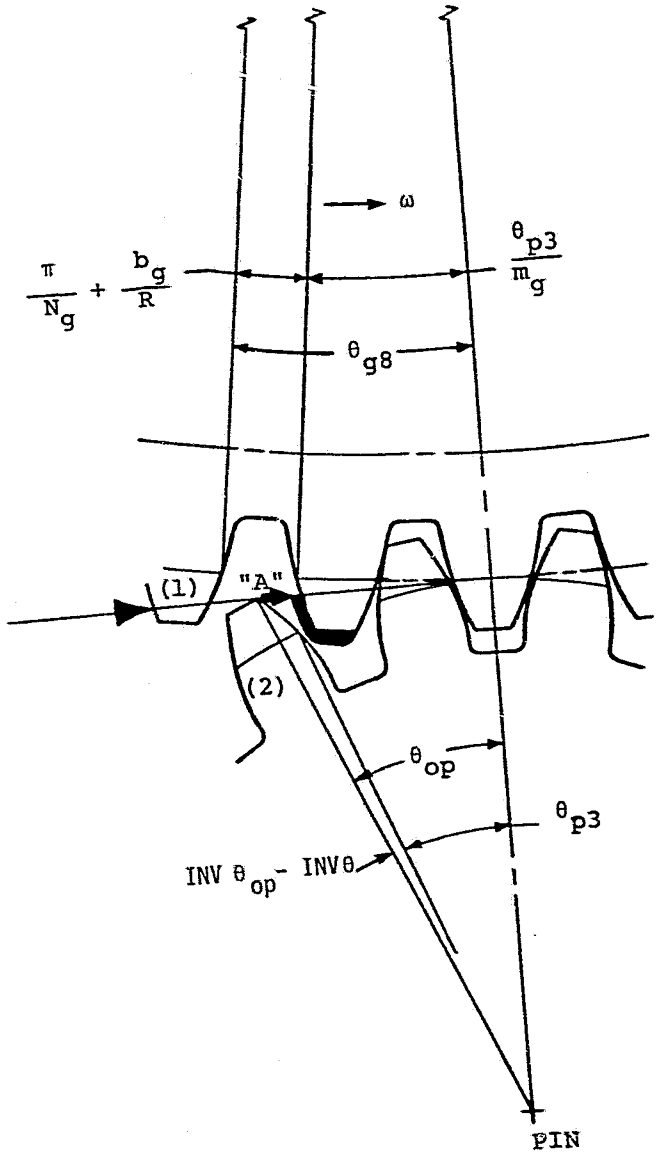

Figure 6.

ORIGINAL PAGE IS OF POOR QUALITY

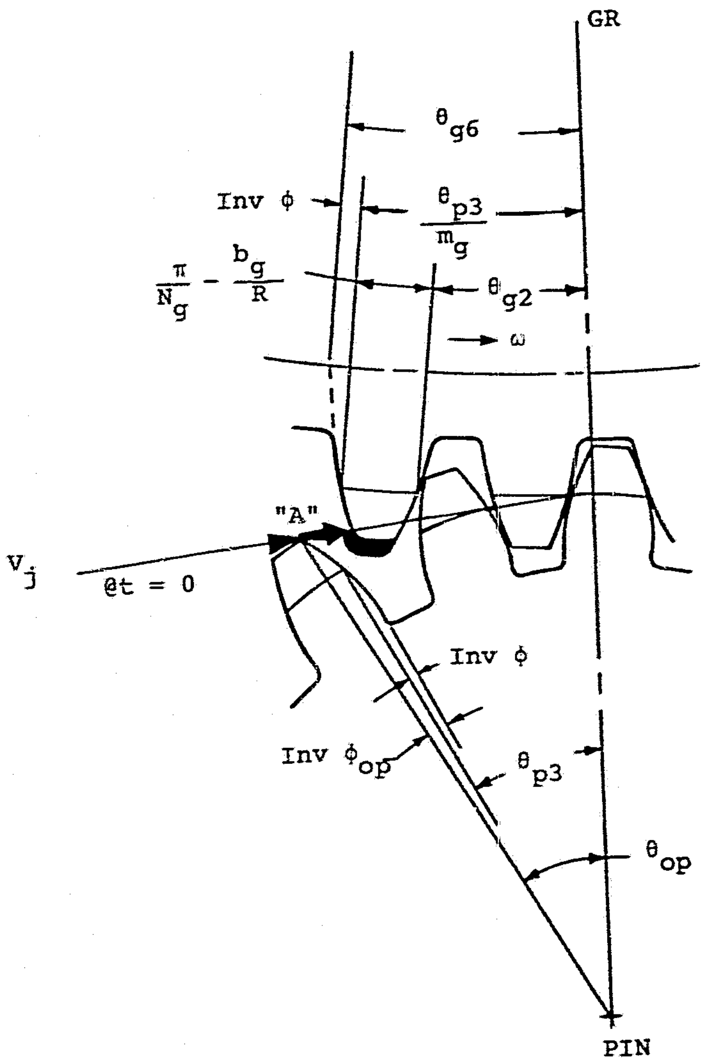
GR
θ_{g6}
Inv ϕ
$\frac{\theta_{p3}}{m_g}$
$\frac{\pi}{N_g} - \frac{b_g}{R}$
θ_{g2}
ω
"A"
V_j
@t = 0
Inv ϕ
Inv ϕ_{op}
θ_{p3}
θ_{op}
PIN

Figure 7.

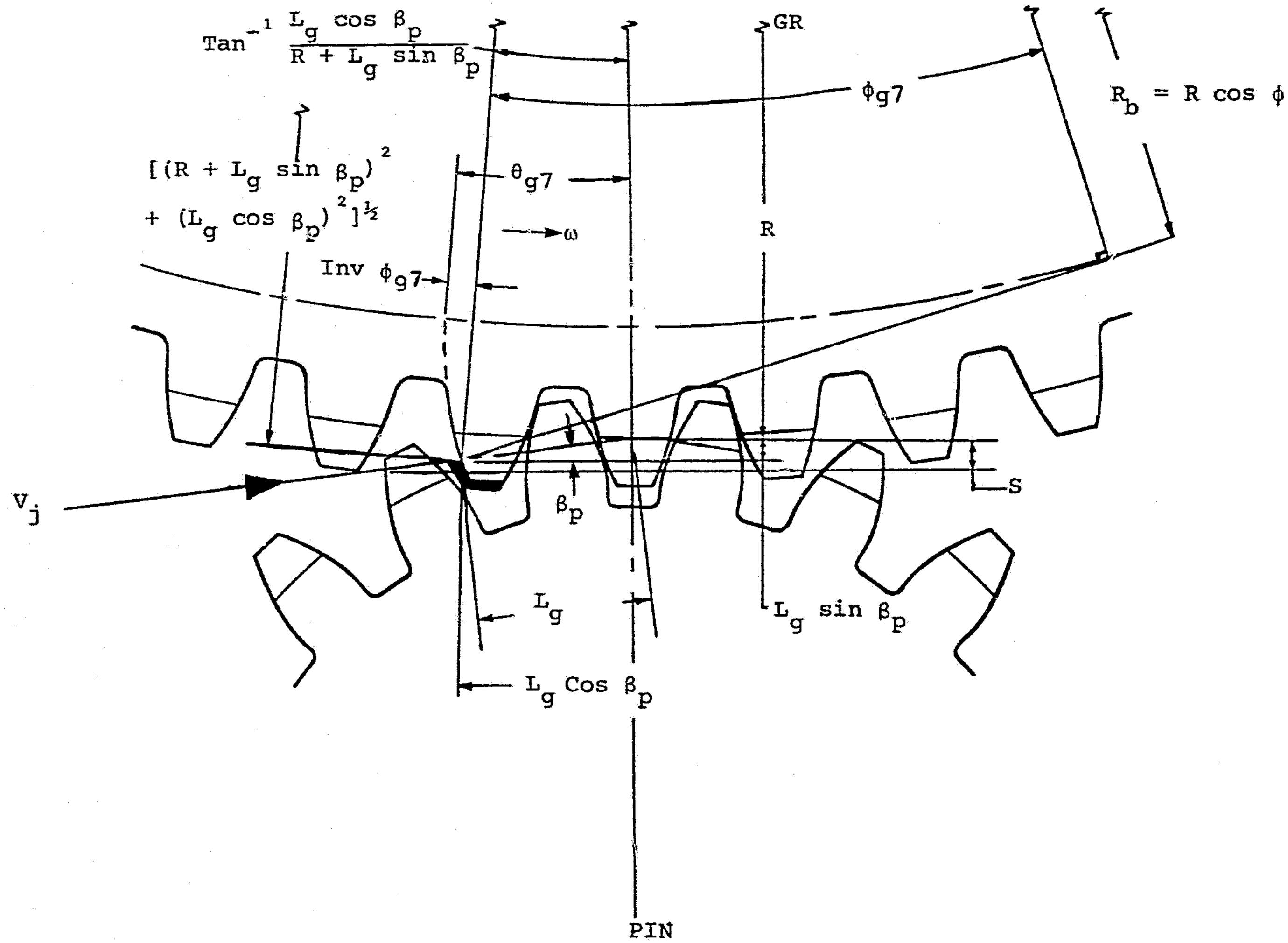
Tan⁻¹ (Lg cos βp)/(R + Lg sin βp)
GR
φg7
Rb = R cos φ
[(R + Lg sin βp)²
+ (Lg cos βp)²]½
θg7
ω
R
Inv φg7
Vj
βp
S
Lg
Lg sin βp
Lg Cos βp
PIN

Figure 8.